Closing today: 3.4(part 1),
Closing Mon: 3.4(part 2)
Closing Wed: 10.2
Closing next Fri: 3.5(part 1)
Office hours today
Fri: 1:30-3:30pm, COM B-006

Motivation/review:

Given y = f(x), we have learned

1. $\frac{dy}{dx} = f'(x) =$ slope of tangent.
2. Equation for tangent:
   $y = f'(a)(x - a) + f(a)$
3. If y = distance and x = time, then is $f'(x) =$ velocity.

Original	*Derivative*
Horiz. Tangent	Zero (f'(x) = 0)
Increasing	Positive
Decreasing	Negative
Vertical Tangent	Undefined

10.2 Parametric Calculus

Recall: Parametric equations describe motion in 2D (or 3D) by giving equations for *x* and *y* separately in terms of time, *t*:

x = x(t), y = y(t)

1. $\frac{dx}{dt} = x'(t) =$ horizontal velocity
2. $\frac{dy}{dt} = y'(t) =$ vertical velocity
3. x = distance, y = distance, t = time
4. $\frac{dy}{dx} = ???$ (we will see this today)

From the worksheet you saw:

Original	*Derivatives*
Horiz. Tangent	y'(t) = 0
Moving Upward	y'(t) positive
Moving Down	y'(t) negative
Vert. Tangent	x'(t) = 0
Moving Right	x'(t) positive
Moving Left	x'(t) negative

Example:

$$x(t) = \frac{1}{2}t, \qquad y(t) = t^2 + 10t$$

"Proof sketch" of fact that $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Assume $x = x(t), y = y(t)$ describes motion along the curve $y = f(x)$.

Then at all times $y(t) = f(x(t))$.

By the chain rule: $y'(t) = f'\big(x(t)\big)x'(t)$.

Therefore, $\frac{y'(t)}{x'(t)} = f'\big(x(t)\big)$.

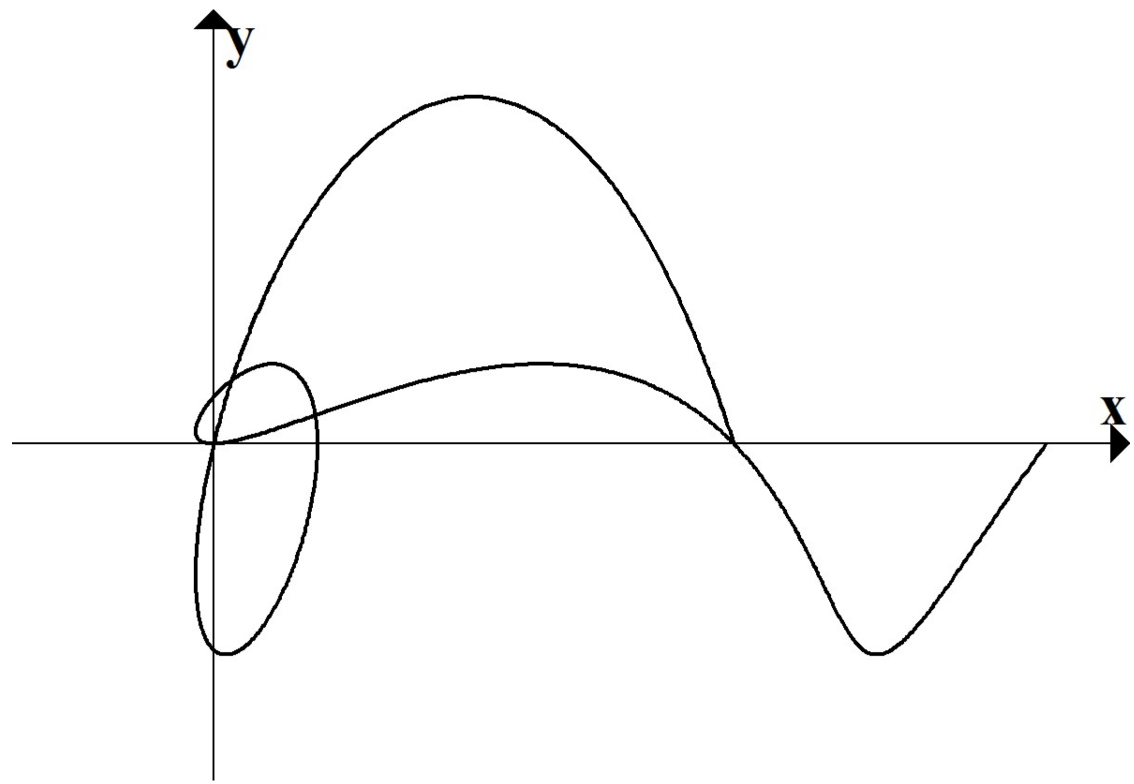
Old Final Question

A particle is moving in the xy-plane according to the equations:

$$x(t) = \cos(\pi t) + t^2 \qquad y(t) = 2(t-1)\sin((t+1)\pi)$$

(a) Find the equation for the tangent line when t = -1.

(b) The particle passes through the origin when t = -1.
Find the next time the particle passes through the origin.



"Proof sketch" that

$$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

Assume $x = x(t), y = y(t)$ describes motion along a curve.

"average speed from *t* to *t+h*" $= \dfrac{\text{change in distance}}{\text{change in time}}$

$$\approx \frac{\sqrt{\big(x(t+h) - x(t)\big)^2 + \big(y(t+h) - y(t)\big)^2}}{h}$$

$$= \sqrt{\left(\frac{x(t+h) - x(t)}{h}\right)^2 + \left(\frac{y(t+h) - y(t)}{h}\right)^2}$$

"instantaneous speed at *t*" is the limit of the above expressions as $h \to 0$

Special parametric equations:

1. An object moving around a circle at a constant speed:

$$(x_c, y_c) = \text{center of circle}$$
$$r = \text{radius}, \theta_0 = \text{initial angle}$$
$$\omega = \text{angular speed } \left(\frac{\text{rad}}{\text{time}}\right)$$
$$\boldsymbol{x = x_c + r\cos(\theta_0 + \omega t)}$$
$$\boldsymbol{y = y_c + r\sin(\theta_0 + \omega t)}$$

Note also the fundamental facts about circular motion (which are only true in radians):

$$\text{linear speed} = v = \omega r,$$
$$\text{arc length} = s = r\theta$$

2. An object moving on a straight line at a constant speed:

$$(x_0, y_0) = \text{initial location}$$
$$a = \text{horizontal velocity}$$
$$b = \text{vertical velocity}$$
$$\boldsymbol{x = x_0 + at}$$
$$\boldsymbol{y = y_0 + bt}$$

Given an applied problem that involves either of these situations, you should initially plug all your information in and solve for the constants.

Directly from homework:

A 4-centimeter rod is attached at one end A to a point on a wheel of radius 2 cm. The other end B is free to move back and forth along a horizontal bar that goes through the center of the wheel. At time *t=0* the rod is situated as in the diagram at the left below. The wheel rotates counterclockwise at 3.5 rev/sec. Thus, when *t=1/21* sec, the rod is situated as in the diagram at the right below.